

Dark radiation and small-scale structure problems with decaying particles

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Although the standard Λ CDM model describes the cosmic microwave background radiation and the large scale structure of the Universe with great success, it has some tensions with observations in the effective number of neutrino species (dark radiation) and the number of small scale structures (overabundance problem). Here we propose a scenario which can relax both tensions through late decays of heavy particle. Supersymmetric axion model is found to be a natural set up for this scenario.

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I. INTRODUCTION

The standard Λ CDM cosmological model has been extremely successful in explaining the observed acoustic peak in the cosmic microwave background (CMB) radiation and the formation of large scale structures (LSS). Despite its success, the Λ CDM model seems to have some tensions with observations at small scales. The measurement of the temperature anisotropy of the CMB showed less power spectrum at small scales, suggesting that the number of effective neutrino species, N_{eff} , has a bigger value than the one predicted by the standard model of particle physics, so the existence of ‘dark radiation’. Another difficulty of Λ CDM model is faced at small scales of the structure formation. N-body simulation with cold dark matter (CDM) has shown a tension with observation in the nonlinear regime of structure formation, producing more substructures in Milky-Way galaxy size than the observed ones.

In the standard cosmological scenario, the thermal plasma after the electron-positron annihilation contains photons and neutrinos. At this epoch, total radiation energy density can be parameterized as

$$\rho_{\text{rad}} = \left[1 + N_{\text{eff}} \frac{7}{8} \left(\frac{T_\nu}{T_\gamma} \right)^4 \right] \rho_\gamma, \quad (1)$$

where $\rho_\gamma = (\pi^2/15)T_\gamma^4$ is the photon energy density, $T_\nu/T_\gamma = (4/11)^{1/3} \simeq 1.40$ after the electron-positron annihilation, and N_{eff} is the effective number of neutrinos, including the contribution from dark radiation if there exists any. In the standard model with three neutrino flavors, the residual heating of the neutrino fluid due to the electron-positron annihilation slightly increases N_{eff} , yielding $N_{\text{eff}}^{\text{SM}} = 3.046$ [1]. However, the WMAP collaboration reported $N_{\text{eff}} = 4.34^{+0.86}_{-0.88}$ (68%CL) through

the measurements of Hubble constant and baryon acoustic oscillation [2]. Similarly higher values of N_{eff} are observed by Atacama Cosmology Telescope (ACT) and South Pole Telescope (SPT), reporting $N_{\text{eff}} = 4.56 \pm 0.75$ [3] and $N_{\text{eff}} = 3.86 \pm 0.42$ [4], respectively. It is expected that the Planck satellite will be able to measure N_{eff} with better precision [5], so make the situation more clear.

The N_{eff} measured in the CMB, $N_{\text{eff}}^{\text{CMB}}$, can be compared with the value $N_{\text{eff}}^{\text{BBN}}$ determined by the big bang nucleosynthesis (BBN). Observations of the primordial ^4He abundance provides the best constraint on $N_{\text{eff}}^{\text{BBN}}$. However there is a controversy between different groups about the relic helium abundance, e.g. [6] and [7], while another recent analysis by Mangano and Serpico [8] gives an upper bound $N_{\text{eff}}^{\text{BBN}} \leq 4$ (95%CL). At any rate, a larger value of N_{eff} can be explained by extra relativistic degree of freedom existing at the epoch prior to the recombination. Many models are suggested to explain this dark radiation [9], including the ones considering the decays of heavy particle as the origin of dark radiation [10].

In the N-body simulation with CDM, the structures form hierarchically, with small structures collapsing first and merging into larger and larger bodies. CDM model describes the distribution and correlation of structures very well at large scales, however there is a large discrepancy at small scales between the observed number of satellite galaxies of the Milky Way and the expected number [11]. This tension has brought many questions on the galaxy formation and evolution as well as the properties of dark matter. One possibility is to impose warm nature of dark matter (WDM) instead of coldness [12]. The free-streaming of WDM can reduce the power spectrum at small scales, which would result in smaller number of galactic subhalos [13].

In fact, WDM model with $m_W \simeq 1 - 4$ keV can alleviate the CDM overabundance problems in many respects. It resolves the discrepancy in the bright satellite galaxies [14], solves the excess of predicted faint galaxies at low and high redshifts, as well as the excess of bright galaxies at low redshifts in the galaxy formation [15]. Also it has better agreement in the HI velocity (width) func-

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tion measured in the ALFALFA survey [16], and in the number of Milky Way satellites [17].

In this work, we examine a scenario in which both dark radiation and WDM find a common origin in the decays of heavy particle. As we will see, supersymmetric axion model with relatively light saxion and axino masses provides a natural set up realizing such scenario. Decays of massive saxion produce (nearly) massless axion pairs and massive axino pairs with different branching ratios. Axions then contribute to the dark radiation, while axinos become warm dark matter which have large velocities to solve the small scale structure problems.

II. DARK RADIATION AND WARM DARK MATTER FROM PARTICLE DECAYS

Let us consider a non relativistic particle X , which decays dominantly to a pair of light particles (DR \equiv dark radiation) and also to a pair of massive particles of mass m (DM \equiv dark matter) with small branching ratio. The energy density of nonrelativistic particles decreases as a^{-3} , while the radiation energy density behaves as a^{-4} . Therefore even when the mass density of X was initially subdominant, it can be important when X decays. After decay, all daughter particles are relativistic, however massive ones become nonrelativistic later due to the redshift of the momentum. After X decays, the resulting dark radiation energy density can be parameterized by the extra effective number of neutrino species $\Delta N_{\text{eff}} \equiv N_{\text{eff}} - N_{\text{eff}}^{\text{SM}}$, which is given by

$$\Delta N_{\text{eff}}(t) = N_{\text{eff}}^{\text{SM}} \frac{\rho_{\text{DR}}(t)}{\rho_{\nu}(t)} = \left(\frac{8}{7}\right) \left(\frac{11}{4}\right)^{4/3} \frac{\rho_{\text{DR}}(t)}{\rho_{\gamma}(t)}, \quad (2)$$

where $\rho_{\nu} = \frac{7}{8} N_{\text{eff}}^{\text{SM}} T_{\nu}^4$ and ρ_{DR} is the extra relativistic energy density called dark radiation.

Here and in the followings, we use the instantaneous decay approximation, and assume that the branching ratio of the X decay into DM pair is small enough, so the DM density at the time τ_X of X decays is negligible. Then the DR energy density right after τ_X is nearly equal to ρ_X right before τ_X :

$$\rho_{\text{DR}}(\tau_X) = \rho_X(\tau_X) = s(\tau_X) M_X Y_X, \quad (3)$$

where M_X and $Y_X \equiv n_X/s$ are the mass and the abundance of the decaying X . Using the entropy density $s = 2\pi^2/45 g_{*S} T^3$ with $g_{*S} \simeq 3.91$ for $T \lesssim 1$ MeV, one easily finds that ΔN_{eff} at time τ_X is given by

$$\begin{aligned} \Delta N_{\text{eff}}(\tau_X) &= \left(\frac{8}{7}\right) \left(\frac{11}{4}\right)^{4/3} \frac{s(\tau_X)}{\rho_{\gamma}(\tau_X)} M_X Y_X \\ &\simeq 11.5 \left(\frac{1 \text{ keV}}{T_{\gamma}(\tau_X)}\right) \left(\frac{M_X Y_X}{1 \text{ keV}}\right), \end{aligned} \quad (4)$$

where the relation between lifetime and the temperature

in the radiation-dominated epoch is given by

$$\tau_X \simeq \left(\frac{90}{\pi^2 g_*}\right)^{1/2} \frac{M_P}{T_{\gamma}^2} \simeq 2.6 \times 10^6 \left(\frac{1 \text{ keV}}{T_{\gamma}}\right)^2 \text{ sec}, \quad (5)$$

with the reduced Planck mass $M_P = 2.4 \times 10^{18}$ GeV and $g_* \simeq 3.36$. Then Eq. (4) can be reexpressed as

$$\Delta N_{\text{eff}}(\tau_X) \simeq 7.1 \left(\frac{\tau_X}{10^6 \text{ sec}}\right)^{1/2} \left(\frac{M_X Y_X}{1 \text{ keV}}\right). \quad (6)$$

The DM particles produced from the decay of X are relativistic initially. However their momenta are redshifted due to the expansion of the Universe, making them non-relativistic at the epoch

$$t_{\text{NR}} \simeq \left(\frac{M_X}{2m}\right)^2 \tau_X, \quad (7)$$

when the red-shifted momentum becomes comparable to the mass

$$p(t_{\text{NR}}) = \frac{M_X}{2} \left(\frac{\tau_X}{t_{\text{NR}}}\right)^{1/2} \simeq m. \quad (8)$$

After this epoch, the energy density of the DM particles produced by the decays of X decreases more slowly than the radiation energy density, and constitute the non-thermally produced dark matter mass density:

$$\begin{aligned} \Omega_{\text{DM}}^{\text{NTP}} h^2 &= 2 f_m \frac{m}{M_X} \Omega_X h^2 \\ &= 5.4 \times 10^{10} f_m \left(\frac{m}{100 \text{ GeV}}\right) Y_X, \end{aligned} \quad (9)$$

where f_m is the branching fraction of $X \rightarrow \text{DM} + \text{DM}$. Imposing the condition

$$\Omega_{\text{DM}}^{\text{NTP}} h^2 \leq \Omega_{\text{DM}}^{\text{WMAP}} h^2 = 0.11, \quad (10)$$

the mass ratio between m and M_X is constrained as

$$f_m \frac{m}{M_X} \leq 2 \times 10^{-4} \left(\frac{1 \text{ keV}}{M_X Y_X}\right), \quad (11)$$

or equivalently (using Eq. (6))

$$f_m \frac{m}{M_X} \leq 1.4 \times 10^{-3} \frac{1}{\Delta N_{\text{eff}}(\tau_X)} \left(\frac{\tau_X}{10^6 \text{ sec}}\right)^{1/2}. \quad (12)$$

In figure 1, we show the contour plot of $f_m = 0.1, 0.01, 0.001$ (blue sold lines) in the plane of m/M_X and τ_X which gives $\Delta N_{\text{eff}}(\tau_X) = 1$ assuming that most of the dark matters are produced from the decay of X .

Then, using Eq. (7) we find the time when DM becomes non relativistic is given by

$$t_{\text{NR}} \geq 12.8 \times 10^{10} (f_m \Delta N_{\text{eff}}(\tau_X))^2 \text{ sec}, \quad (13)$$

and therefore the DM particles produced by the decays of X can have large kinetic energy which can erase the

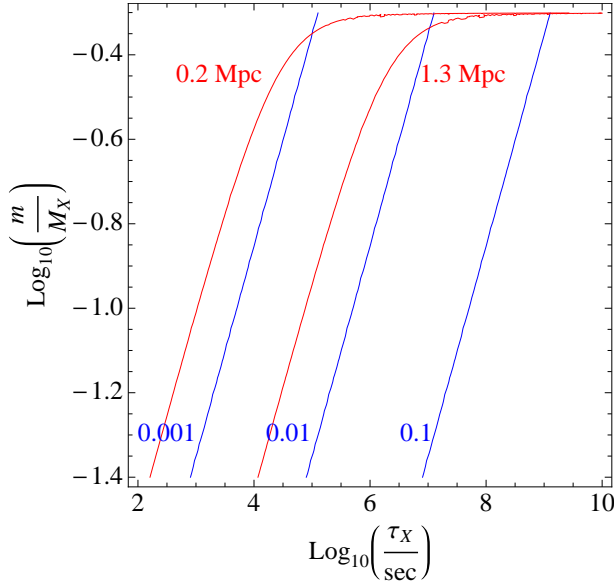


FIG. 1: Blue solid lines show the contour plot of $c_m f_m = 0.1, 0.01, 0.001$ which gives $\Delta N_{\text{eff}}(\tau_X) = 1$ when dark matters are produced from the decay of X (equality in Eq. (12)). Red solid lines are the contour plot of $\lambda_{\text{FS}} = 0.2, 1.3$ Mpc in the plane of m/M_X and τ_X . In the region between the red lines and $f_m \sim 0.001$, both the dark radiation and small scale structure problems can be explained with the decay of particles.

small scale structure formation. The characteristic free-streaming length is given by

$$\lambda_{\text{FS}} = \int_{\tau_X}^{\tau_{\text{eq}}} \frac{v(t)}{a(t)} dt, \quad (14)$$

which can be approximated as

$$\lambda_{\text{FS}} \simeq 1.0 \text{ Mpc} \left(\frac{u_\tau^2 \tau_X}{10^6 \text{ sec}} \right)^{1/2} \left[1 - 0.07 \ln \left(\frac{u_\tau^2 \tau_X}{10^6 \text{ sec}} \right) \right], \quad (15)$$

where u_τ is evaluated at τ_X and expressed as

$$u_\tau \equiv \frac{|\vec{p}|}{m} \simeq \frac{M_X}{2m} \left(1 - 4 \frac{m^2}{M_X^2} \right). \quad (16)$$

The characteristic free-streaming length can be related to the thermally produced warm dark matter mass via [18]

$$\lambda_f \equiv \frac{2\pi}{k_f} = 1.29 \left(\frac{\Omega_m h^2}{0.11} \right)^{1/3} \left(\frac{m_W}{\text{keV}} \right)^{-4/3} \text{ Mpc}. \quad (17)$$

The Lyman- α forest data constrains the cut-off scale of the power spectrum. In terms of the warm dark matter mass m_W , it has been claimed to give a 2σ -bound $m_W > 2 \text{ keV}$ [19, 20], however it can be relaxed to $m_W > 0.9 \text{ keV}$ if the less reliable data are rejected [19, 21, 22]. Considering the blazer heating, the

revised bound $m_W > 1.7 \text{ keV}$ [23] can have about 30% systematic uncertainty [24]. Therefore here we adopt WDM with $m_W = 1 - 4 \text{ keV}$ as a solution for the small scale structure formation, which can be consistent with the Lyman- α constraint. This corresponds to the free streaming scale $\lambda_{\text{FS}} = 0.2 - 1.3 \text{ Mpc}$. In Fig. 1, we show the contour plot of $\lambda_{\text{FS}} = 0.2, 1.3 \text{ Mpc}$ in the plane of τ_X and m/M_X .

III. SUPERSYMMETRIC AXION MODEL

Supersymmetric axion model provides a viable example which would realize the scenario discussed in the previous section. The model includes a $U(1)_{\text{PQ}}$ symmetry spontaneously broken by the vacuum expectation value of a PQ charged but SM neutral scalar field ϕ . We can identify the radial component of ϕ as the saxion s , and the phase component as the axion a . If there exists $SU(3)_C$ charged fermion which transforms under $U(1)_{\text{PQ}}$, the associated a becomes the QCD axion solving the strong CP problem [25]. In supersymmetric model, ϕ can be considered as a chiral superfield given by

$$S = \left(F_a + \frac{s}{\sqrt{2}} \right) \exp \left(\frac{ia}{\sqrt{2}F_a} \right) + \sqrt{2}\theta\tilde{a} + \theta^2 F^S, \quad (18)$$

where $F_a = \langle \phi \rangle$ is the axion decay constant, and \tilde{a} is the axino, the fermionic superpartner of axion. While the axion gets a mass only by QCD anomaly, the saxion and axino can be much heavier than the axion due to supersymmetry (SUSY) breaking effects. Because the interactions of the axion supermultiplet are suppressed by the axion decay constant, one can easily find a setup which would explain the dark radiation and small scale problems with saxion decaying mostly to axions for dark radiation and also to axinos with small branching ratio for warm dark matter.

The relevant Lagrangian for our discussion is as follows.

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}(\partial_\mu s)^2 + \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}\tilde{a} i\sigma^\mu \partial_\mu \tilde{a} \\ & - \frac{1}{2}m_s^2 s^2 - \frac{1}{2}(m\tilde{a}\tilde{a} + h.c.) \\ & + \frac{s}{\sqrt{2}F_a} \left[(\partial_\mu a)^2 + \frac{\lambda}{2}(m\tilde{a}\tilde{a} + h.c.) + \sum_A \frac{g_A^2 C_A}{32\pi^2} F^{A\mu\nu} F_{\mu\nu}^A \right]. \end{aligned} \quad (19)$$

Here m_s and m denote the saxion and axino masses, respectively, and λ and C_A are model-dependent parameters of order unity or smaller than one. For the KSVZ type axion model [26], the above terms are enough, while for the DFSZ type axion model [27], saxion can have sizable couplings to the SM fermions which are charged under $U(1)_{\text{PQ}}$. Here we take the KSVZ type model as our example, in which the SM fermions are not PQ charged, and so their couplings to saxion and axion can be ignored.

In the limit $m_s \gg m$, saxions decay dominantly to axion pairs with a decay rate

$$\Gamma(s \rightarrow 2a) \simeq \frac{1}{64\pi} \frac{m_s^3}{F_a^2}, \quad (20)$$

yielding the saxion lifetime

$$\tau_s \simeq 1.3 \times 10^5 \left(\frac{m_s}{100 \text{ MeV}} \right)^{-3} \left(\frac{F_a}{10^{12} \text{ GeV}} \right)^2 \text{ sec}. \quad (21)$$

The saxion field is initially displaced from the present vacuum value, and starts oscillation at the moment when the expansion rate $H \sim m_s$. If it happens before the reheating after inflation, the energy density to entropy density ratio, which is constant during the radiation-dominated epoch, is given by

$$\frac{\rho_s}{s} = m_s Y_s = 2.2 \times 10^{-8} \left(\frac{T_R}{10^6 \text{ GeV}} \right) \left(\frac{F_a}{10^{12} \text{ GeV}} \right)^2 \text{ GeV}, \quad (22)$$

where we used the initial displacement of the saxion $\delta s \simeq F_a$. We then find from Eq. (6) that ΔN_{eff} at the time of saxion decay is given by

$$\Delta N_{\text{eff}} = 0.056 \left(\frac{100 \text{ MeV}}{m_s} \right)^{3/2} \left(\frac{F_a}{10^{12} \text{ GeV}} \right)^3 \left(\frac{T_R}{10^6 \text{ GeV}} \right). \quad (23)$$

The saxion also produces axinos with decay rate

$$\Gamma(s \rightarrow \tilde{a}\tilde{a}) = \frac{\lambda^2 m^2 m_s}{32\pi F_a^2} \left[1 - \left(\frac{4m^2}{m_s^2} \right) \right]^{3/2}, \quad (24)$$

for which the branching ratio is given by

$$f_m \simeq \left(\frac{2\lambda^2 m^2}{m_s^2} \right) \left(1 - \frac{4m^2}{m_s^2} \right)^{3/2}. \quad (25)$$

Such non-thermally produced axinos play the role of warm dark matter and can solve the small scale structure problems as explained in the previous section.

In Fig. 2, we show the viable region in the plane of m_s and F_a for $m/m_s = 0.25$ and the reheat temperature $T_R = 5 \times 10^5 \text{ GeV}$ of the primordial inflation. The blue lines denote $\Delta N_{\text{eff}} = 0.5$ and 1.5, while the red lines stand for $\lambda_{\text{FS}} = 0.2$ and 1.3 Mpc. On the dashed Magenta line, axinos produced by saxion decays constitute most of the dark matter for $\lambda = 0.1$ and 0.2. Therefore in the overlapped region of blue and red bands, which corresponds to $100 \text{ MeV} \lesssim m_s \lesssim 1 \text{ GeV}$ and $3 \times 10^{12} \text{ GeV} \lesssim F_a \lesssim 10^{13} \text{ GeV}$, both dark radiation and small scale structure problems can be explained with corresponding value of λ between 0.1 and 0.2 respectively. In this region the relic density of the thermally produced axinos is less than about 10 % of dark matter (black solid line), thus most of the axino dark matters are produced from saxion decays.

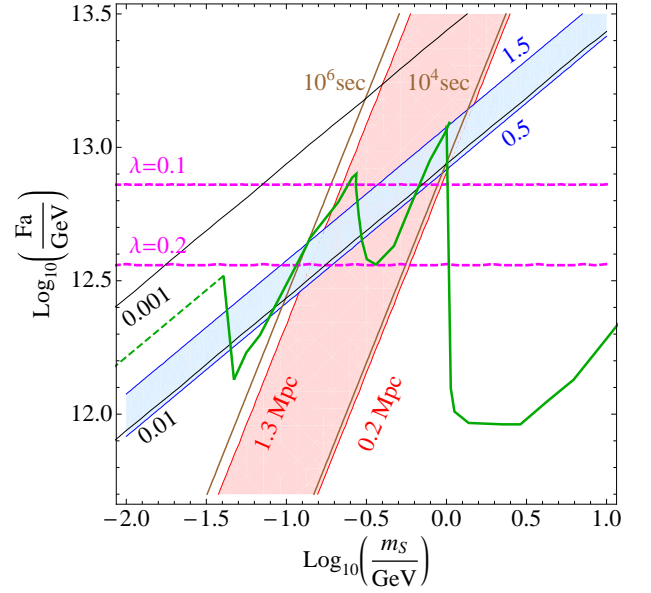


FIG. 2: Contour plots of ΔN_{eff} and λ_{FS} in the (m_s, F_a) plane with other cosmological constraints. Here we used $T_R = 5 \times 10^5 \text{ GeV}$, $m/m_s = 0.25$. Blue lines denote $\Delta N_{\text{eff}} = 0.5, 1.5$. Red lines show $\lambda_{\text{FS}} = 0.2, 1.3 \text{ Mpc}$. Black lines correspond to thermal production of the axino $\Omega_a^{\text{TP}} h^2 = 0.001, 0.01$. Brown lines denote the lifetime of the saxion $10^4 \text{ sec}, 10^6 \text{ sec}$. The horizontal magenta (dotted) lines represent $\Omega_a^{\text{NTP}} h^2 = 0.1$ for $\lambda = 0.1, 0.2$ respectively. The green line shows the BBN (solid) and CMB (dashed) constraint and the lower region is allowed.

The decay of saxions can produce electromagnetic and hadronic particles which can disrupt the light element abundances after BBN. The lifetime of saxion in the region of our interest is between 10^4 and 10^6 sec . In Fig. 2, we show the BBN and CMB bound with green lines [28] and the upper region is disallowed. In this region, the constraints on the hadronic and electromagnetic injections lead to

$$B_h \frac{\rho_s}{s} \lesssim 10^{-14} \text{ GeV}, \quad B_{\text{em}} \frac{\rho_s}{s} \lesssim 10^{-6} - 10^{-13} \text{ GeV}, \quad (26)$$

where B_h and B_{em} denote the branching ratios for the hadronic and electromagnetic injections. These constraints can be easily satisfied if the saxion mass is below the pion production threshold $m_s < 2m_\pi$.

We then find that the saxion mass around 100 MeV with Peccei-Quinn scale around $5 \times 10^{12} \text{ GeV}$ and reheating temperature $5 \times 10^5 \text{ GeV}$ can explain both dark radiation and small scale structure problems, while satisfying all the constraints from BBN, cold axino abundance, and gravitino problem. If the axion in our model is a QCD axion solving the strong CP problem, we need a small axion misalignment angle $\theta_i \lesssim 0.1$ in order for the cold axion dark matter to be subdominant compared to the warm axino dark matter produced by saxion decays.

We close this section with a brief discussion of SUSY breaking schemes which can give the saxion mass $m_s \sim 100$ MeV and the axino mass $m \sim 0.2m_s$. We first note that in gauge mediation with a messenger scale $M_{\text{mess}} \ll F_a$, saxion and axino masses can be much lighter than the MSSM soft parameters as they are further suppressed by some powers of M_{mess}/F_a . Furthermore if saxion is stabilized by radiative effects, a small hierarchy between the saxion and axino masses, e.g. $m/m_s \sim \sqrt{1/4\pi^2}$, can arise in a natural manner. On the other hand, if there exists a SUSY breaking sector well sequestered from the visible sector as well as from the PQ sector, the gravitino mass can be much heavier than the axino mass. These points suggest that there can be a plenty of rooms for the mediation of SUSY breaking yielding the desired saxion and axino masses while satisfying the known phenomenological and cosmological constraints. An explicit construction of such model is beyond the scope of this paper, and will be the subject of upcoming work [29].

IV. CONCLUSION

We have examined the possibility that late decays of massive particle after BBN can provide a common origin

for dark radiation around the epoch of recombination and warm dark matter with free streaming which can solve the small scale structure problems. As a specific example, we proposed a supersymmetric axion model in which dark radiation axions and warm dark matter axinos are produced by the decays of saxion, and identified a parameter space which can successfully realize the scenario while satisfying all the cosmological constraints.

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- [1] G. Mangano, G. Miele, S. Pastor, T. Pinto, O. Pisanti and P. D. Serpico, Nucl. Phys. B **729** (2005) 221 [hep-ph/0506164].
 - [2] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **192** (2011) 18 [arXiv:1001.4538 [astro-ph.CO]].
 - [3] J. Dunkley, R. Hlozek, J. Sievers, V. Acquaviva, P. A. R. Ade, P. Aguirre, M. Amiri and J. W. Appel *et al.*, Astrophys. J. **739** (2011) 52 [arXiv:1009.0866 [astro-ph.CO]].
 - [4] R. Keisler, C. L. Reichardt, K. A. Aird, B. A. Benson, L. E. Bleem, J. E. Carlstrom, C. L. Chang and H. M. Cho *et al.*, Astrophys. J. **743** (2011) 28 [arXiv:1105.3182 [astro-ph.CO]].
 - [5] J. Hamann, J. Lesgourgues and G. Mangano, JCAP **0803** (2008) 004 [arXiv:0712.2826 [astro-ph]].
 - [6] Y. I. Izotov and T. X. Thuan, Astrophys. J. **734** (2011) 82 [arXiv:1104.0817 [astro-ph.CO]].
 - [7] E. Aver, K. A. Olive and E. D. Skillman, JCAP **1005** (2010) 003 [arXiv:1001.5218 [astro-ph.CO]].
 - [8] G. Mangano and P. D. Serpico, Phys. Lett. B **701** (2011) 296 [arXiv:1103.1261 [astro-ph.CO]].
 - [9] S. Galli, F. Iocco, G. Bertone and A. Melchiorri, Phys. Rev. D **80** (2009) 023505 [arXiv:0905.0003 [astro-ph.CO]]; J. Zavala, M. Vogelsberger and S. D. M. White, Phys. Rev. D **81** (2010) 083502 [arXiv:0910.5221 [astro-ph.CO]]; J. Hamann, S. Hannestad, G. G. Raffelt, I. Tamborra and Y. Y. Y. Wong, Phys. Rev. Lett. **105** (2010) 181301 [arXiv:1006.5276 [hep-ph]]; K. Nakayama, F. Takahashi and T. T. Yanagida, Phys. Lett. B **697** (2011) 275 [arXiv:1010.5693 [hep-ph]]; Z. Hou, R. Keisler, L. Knox, M. Millea and C. Reichardt, arXiv:1104.2333 [astro-ph.CO]; M. Archidiacono, E. Calabrese and A. Melchiorri, Phys. Rev. D **84** (2011) 123008 [arXiv:1109.2767 [astro-ph.CO]]; M. Blennow, E. Fernandez-Martinez, O. Mena, J. Redondo and P. Serra, JCAP **1207** (2012) 022 [arXiv:1203.5803 [hep-ph]]; O. E. Bjaelde, S. Das and A. Moss, arXiv:1205.0553 [astro-ph.CO]; E. Calabrese, M. Archidiacono, A. Melchiorri and B. Ratra, arXiv:1205.6753 [astro-ph.CO]; M. Archidiacono, E. Giusarma, A. Melchiorri and O. Mena, arXiv:1206.0109 [astro-ph.CO]; C. Boehm, M. J. Dolan and C. McCabe, arXiv:1207.0497 [astro-ph.CO].
 - [10] R. J. Scherrer and M. S. Turner, Astrophys. J. **331** (1988) 19 [Astrophys. J. **331** (1988) 33]; E. J. Chun, D. Comelli and D. H. Lyth, Phys. Rev. D **62** (2000) 095013 [hep-ph/0008133]; K. Ichikawa, M. Kawasaki, K. Nakayama, M. Senami and F. Takahashi, JCAP **0705** (2007) 008 [hep-ph/0703034 [HEP-PH]]; W. Fischler and J. Meyers, Phys. Rev. D **83** (2011) 063520 [arXiv:1011.3501 [astro-ph.CO]]; J. L. Menestrina and R. J. Scherrer, Phys. Rev. D **85** (2012) 047301 [arXiv:1111.0605 [astro-ph.CO]]; J. Hasenkamp, Phys. Lett. B **707** (2012) 121 [arXiv:1107.4319 [hep-ph]]; T. Kobayashi, F. Takahashi, T. Takahashi and M. Yamaguchi, JCAP **1203** (2012) 036 [arXiv:1111.1336 [astro-ph.CO]]; D. Hooper, F. S. Queiroz and N. Y. Gnedin, Phys. Rev. D **85** (2012) 063513 [arXiv:1111.6599 [astro-ph.CO]]; K. S. Jeong and F. Takahashi, arXiv:1201.4816 [hep-ph]; R. Jinno, T. Moroi and K. Nakayama, arXiv:1208.0184 [astro-ph.CO].
 - [11] A. A. Klypin, A. V. Kravtsov, O. Valenzuela and F. Prada, Astrophys. J. **522** (1999) 82 [astro-ph/9901240]; B. Moore, S. Ghigna, F. Governato, G. Lake, T. R. Quinn, J. Stadel and P. Tozzi, Astrophys.

- J. **524** (1999) L19 [astro-ph/9907411].
- [12] P. Bode, J. P. Ostriker and N. Turok, *Astrophys. J.* **556** (2001) 93 [astro-ph/0010389].
- [13] S. Borgani, A. Masiero and M. Yamaguchi, *Phys. Lett. B* **386** (1996) 189 [hep-ph/9605222]; W. B. Lin, D. H. Huang, X. Zhang and R. H. Brandenberger, *Phys. Rev. Lett.* **86** (2001) 954 [astro-ph/0009003]; J. Hisano, K. Kohri and M. M. Nojiri, *Phys. Lett. B* **505** (2001) 169 [hep-ph/0011216]; J. A. R. Cembranos, J. L. Feng, A. Rajaraman and F. Takayama, *Phys. Rev. Lett.* **95** (2005) 181301 [hep-ph/0507150]; M. Kaplinghat, *Phys. Rev. D* **72** (2005) 063510 [astro-ph/0507300]; K. Jedamzik, M. Lemoine and G. Moulata, *JCAP* **0607** (2006) 010 [astro-ph/0508141]; F. Borzumati, T. Bringmann and P. Ullio, *Phys. Rev. D* **77** (2008) 063514 [hep-ph/0701007]; K. -Y. Choi, J. E. Kim, H. M. Lee and O. Seto, *Phys. Rev. D* **77** (2008) 123501 [arXiv:0801.0491 [hep-ph]]; S. Bailly, K. -Y. Choi, K. Jedamzik and L. Roszkowski, *JHEP* **0905** (2009) 103 [arXiv:0903.3974 [hep-ph]]; K. -Y. Choi, L. Covi, J. E. Kim and L. Roszkowski, *JHEP* **1204** (2012) 106 [arXiv:1108.2282 [hep-ph]]; J. L. Feng, Z. 'e. Surujon and H. -B. Yu, arXiv:1205.6480 [hep-ph].
- [14] M. R. Lovell, V. Eke, C. S. Frenk, L. Gao, A. Jenkins, T. Theuns, J. Wang and D. M. White *et al.*, *Mon. Not. Roy. Astron. Soc.* **420** (2012) 2318 [arXiv:1104.2929 [astro-ph.CO]].
- [15] N. Menci, F. Fiore and A. Lamastra, arXiv:1201.1617 [astro-ph.CO]; A. V. Maccio and F. Fontanot, arXiv:0910.2460 [astro-ph.CO].
- [16] E. Papastergis, A. M. Martin, R. Giovanelli and M. P. Haynes, *Astrophys. J.* **739** (2011) 38 [arXiv:1106.0710 [astro-ph.CO]]. J. Zavala, Y. P. Jing, A. Faltenbacher, G. Yepes, Y. Hoffman, S. Gottlober and B. Catinella, *Astrophys. J.* **700** (2009) 1779 [arXiv:0906.0585 [astro-ph.CO]].
- [17] E. Polisensky and M. Ricotti, *Phys. Rev. D* **83** (2011) 043506 [arXiv:1004.1459 [astro-ph.CO]].
- [18] J. Sommer-Larsen and A. Dolgov, *Astrophys. J.* **551** (2001) 608 [astro-ph/9912166].
- [19] M. Viel, J. Lesgourgues, M. G. Haehnelt, S. Matarrese and A. Riotto, *Phys. Rev. Lett.* **97** (2006) 071301 [astro-ph/0605706].
- [20] U. Seljak, A. Makarov, P. McDonald and H. Trac, *Phys. Rev. Lett.* **97** (2006) 191303 [astro-ph/0602430].
- [21] L. G. v. d. Aarssen, T. Bringmann and C. Pfrommer, arXiv:1205.5809 [astro-ph.CO].
- [22] A. Boyarsky, J. Lesgourgues, O. Ruchayskiy and M. Viel, *JCAP* **0905** (2009) 012 [arXiv:0812.0010 [astro-ph]].
- [23] A. Boyarsky, O. Ruchayskiy and D. Iakubovskiy, *JCAP* **0903** (2009) 005 [arXiv:0808.3902 [hep-ph]].
- [24] E. Puchwein, C. Pfrommer, V. Springel, A. E. Broderick and P. Chang, arXiv:1107.3837 [astro-ph.CO].
- [25] J. E. Kim and G. Carosi, *Rev. Mod. Phys.* **82** (2010) 557 [arXiv:0807.3125 [hep-ph]].
- [26] J. E. Kim, *Phys. Rev. Lett.* **43** (1979) 103; M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, *Nucl. Phys. B* **166** (1980) 493.
- [27] M. Dine, W. Fischler and M. Srednicki, *Phys. Lett. B* **104** (1981) 199; A. P. Zhitnitskii, *Sov. J. Nucl. Phys.* **31**, 260 (1980).
- [28] K. Jedamzik, *Phys. Rev. D* **70** (2004) 063524 [astro-ph/0402344]; M. Kawasaki, K. Kohri and T. Moroi, *Phys. Lett. B* **625** (2005) 7 [astro-ph/0402490]; M. Kawasaki, K. Kohri and T. Moroi, *Phys. Rev. D* **71** (2005) 083502 [astro-ph/0408426].
- [29] K. Choi, K.-Y. Choi, C.S. Shin, in preparation.